On the Performance Analysis of Multi-antenna Relaying System over Rayleigh Fading Channel

Anup Dey[1], Pradip Kumar Ghosh[2], Kapil Gupta[2]

[1]Electronics and Communication Engineering Department,
Kalyani Government Engineering College,
Kalyani, India.
e-mail: a_dey2002@yahoo.com

[2]Electronics and Communication Engineering Department,
Mody Institute of Technology and Science,
Lakshmangarh, Sikar, India.
e-mail: pkghosh_ece@yahoo.co.in, kapil_mbm@rediffmail.com

Abstract—**In this work, the end-to-end performance of an amplify-and-forward multi-antenna infrastructure-based relay (fixed relay) system over flat Rayleigh fading channel is investigated. New closed form expressions for the statistics of the received signal-to-noise ratio (SNR) are presented and applied for studying the outage probability and the average bit error rate of the digital receivers. The results reveal that the system performance improves significantly (roughly 3 dB) for M=2 over that for M=1 in both low and high signal-to-noise ratio. However, little additional performance improvement can be achieved for M>2 relative to M=2 at high SNR.**

Index Terms — **Diversity, fading channel, moment generating function(mgf) , outage probability.**

I. INTRODUCTION

Relaying is used primarily for extending the coverage area of a transmitter. Recently it has gained considerable interest due to its possible application in cooperative diversity [1]. Cooperative diversity emerged as a promising technique due to its ability to combat deep fade in a wireless channel and uses single or multiple relay links to achieve spatial diversity forming a virtual antenna array at the receiver. Applications of cooperative diversity include future cellular and ad-hoc wireless communication systems [2-4].

The end-to-end performances of a two-hop relayed transmission link is studied analytically in [1] over Rayleigh fading channel where only a single antenna located at the source and destination nodes and a single antenna each for reception and transmission at the relay. In recent years, realizing the potential benefit of MIMO (multiple-input-multiple-output) systems over the single antenna system , accommodating multiple antenna at the relay node is gaining great interest [5, and references there in] .However, deployment of multiple antennas at the mobile terminals often encounters various implementation problems as the future wireless terminals are expected to be small and light. In contrast to mobile terminals, accommodating a small number of antennas on

infrastructure-based fixed relays is feasible [6], and the single antenna relay can be considered as a special case of this set-up. In [6] the end-to-end performance of such fixed multi-antenna relay(infrastructure-based relaying) is studied for the decode-and forward(DF) relaying scheme(more specifically threshold DF) and examined the achievable cooperative diversity. In [7], the performance of selection combining (SC) based multi-antenna fixed relay for both amplify-and-forward and decode-and-forward relaying is presented.

In this paper, we investigate the end-to-end performance of maximum ratio combining (MRC) based-multi-antenna fixed relay (infrastructure-based relay) system with amplify-and-forward (AF) relaying technique which offers a simpler hardware circuitry for its implementation compare to the decode-and forward(DF) relaying. New closed form expressions for the statistics of the received signal-to-noise ratio (SNR) are developed for independent flat Rayleigh fading channel. More specifically, the probability density function (pdf), the cumulative density function (cdf), the moment generating function (mgf) and the output SNR moments are derived. These statistical results are the applied to study the important performance metrics of the system. Outage Probability (OP), received SNR moments and the average bit error rates (ABERs) for binary differential phase shift keying and binary frequency shift keying are also derived in closed form.

The remainder of the paper is organized as follows. In section II, the infrastructure based relaying system and channel models are introduced and pdf cdf and mgf of the received SNR are derived. Section III provides the expressions for various performance metrics of the system, while in section IV, the results of these metrics are applied and analyzed. Finally, concluding remarks are provided in section V.
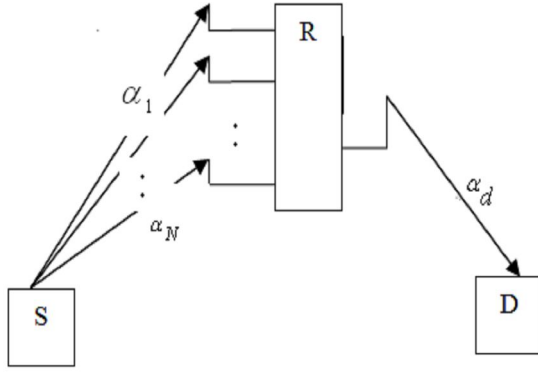
Figure 1. A wireless communication system where terminal R is equipped with M diversity antennas at the input and a single diversity antenna at the output and relaying the signal from terminal S to terminal D.

II. MULTI-ANTENNA (FIXED RELAY) SYSTEM

A. System and Channel Model

Figure 1 shows a two-hop wireless communication system where terminal S is communicating with terminal D through the relay terminal R. The relay R is equipped with M receiving antennas for reception of signal transmitted from terminal S and a single transmitting antenna to convey the signal to terminal D after suitable amplification at the relay node. We assume that maximum ratio combining (MRC) is used for receiving the signal at R. In MRC, the branch with the instantaneous SNR is selected as the output of the combiner is given by [8],

$$\gamma_{mrc} = \sum_{i=1}^{M} \gamma_i$$

(4)

where the instantaneous signal-to-noise ratio (SNR)

of the i-th branch is, $\gamma_i = \dfrac{E_s}{N_0}[\alpha_i^2]$, i = 1,2,..,M, with

α_i being the Rayleigh fading amplitude of the channel between terminals S and the antennas (i = 1, 2, M,) at relay R, E_s is the energy of the transmitted signals and N_0 is the one-sided noise power spectral density per branch. We assume the channel is frequency non-selective and slowly varying such that it is constant over the transmitted symbols interval. Noting that α_i is Rayleigh distributed, α_i^2 is the exponentially distributed random variables. If we further assume that $(SNR)_i$ is the same for all diversity branches

(i.e., $\overline{\gamma_1} = \overline{\gamma_2} = .. = \overline{\gamma_M} = \overline{\gamma_s}$), the output SNR of the MRC combiner (or, the SNR between S and R , $\gamma_{s,r}$) follows the Gamma distribution with the probability density function (pdf) given by [8]

$$p_{\Gamma_{s,r}}(\gamma_{s,r}) = \frac{\gamma_{s,r}^{M-1}}{\overline{\gamma_s}^M \Gamma(M)} e^{-\frac{\gamma_{s,r}}{\overline{\gamma_s}}}$$

(5)

Assuming $\alpha_{r,d}$, the fading amplitude of the channel between antennas at R and D as Rayleigh type, and the average SNR being $\overline{\gamma_d}$, by similar reasoning the pdf of $\Gamma_{r,d}$ can be written as

$$p_{\Gamma_{r,d}}(\gamma_{r,d}) = \frac{1}{\overline{\gamma_d}} e^{-\frac{\gamma_{r,d}}{\overline{\gamma_d}}}$$

(6)

Choosing the appropriate gain at the relay terminal before re-transmission, the overall SNR at the receiving terminal D can be very closely upper bounded as, [1],

$$\gamma_{eq} = \frac{\Gamma_{s,r}\Gamma_{r,d}}{\Gamma_{s,r} + \Gamma_{r,d}}$$

$$= \frac{1}{\dfrac{1}{\Gamma_{s,r}} + \dfrac{1}{\Gamma_{r,d}}}$$

(7)

where $\Gamma_{s,r}$ is the output SNR of the MRC, and $\Gamma_{r,d}$ is the instantaneous SNR between R and D. Eq. (7) is often preferred for performance analysis in dual-hop relayed transmission links due to it's mathematical tractability [1].

B. Derivation of the cdf and pdf of received SNR at destination

Let $X = \dfrac{1}{\Gamma_{s,r}}$ and $Y = \dfrac{1}{\Gamma_{r,d}}$, and $\overline{\gamma_s} = \overline{\gamma_d} = \overline{\gamma}$ then the pdfs of X and Y can be evaluated with the help of [9 eq. (5-18)] and [9 eq. (5-20)] resulting

$$p_X(x) = \frac{x^{-M-1}}{\overline{\gamma}^M \Gamma(M)} e^{-\frac{1}{x\overline{\gamma}}} \text{ and } p_Y(y) = \frac{1}{y^2\overline{\gamma}} e^{-\frac{1}{y\overline{\gamma}}} .$$ Now, the mgf of X and Y can be obtained, using [10 eq. (3.471.9) and [10 eq. (8.486.16)]. The results are $M_X(s) = \frac{2}{\Gamma(M)}(\sqrt{\beta s})^M K_M(2\sqrt{\beta s})$, and $M_Y(s) = 2(\sqrt{\beta s})K_1(2\sqrt{\beta s})$, where $\overline{\gamma} = \frac{1}{\beta}$ and $K_v(\cdot)$ is the v-th order modified Bessel function of second kind .

Assuming independence between X and Y, the mgf of $Z = \dfrac{1}{\gamma_{eq}}$ can be written as

$$M_Z(s) = \frac{4}{\Gamma(M)}(\sqrt{\beta s})^{M+1} K_M(2\sqrt{\beta s})K_1(2\sqrt{\beta s})$$

(8)

The cdf of the destination SNR, γ_{eq} may be written as [1],

$$F_{\gamma_{eq}}(\gamma) = 1 - F_Z(z)\Big|_{z=\frac{1}{\gamma}}$$

(9)

where $F_Z(z)$ denotes the cdf of Z. Now, eq. (9) can be rewritten as

$$F_{\gamma_{eq}}(\gamma) = 1 - L^{-1}\left(\frac{M_Z(s)}{s}\right)\Big|_{z=\frac{1}{\gamma}}$$

$$=1-L^{-1}\left(\frac{4}{\Gamma(M)}\beta^{\frac{M+1}{2}}(s)^{\frac{M-1}{2}} K_M(2\sqrt{\beta s})K_1(2\sqrt{\beta s})\right)\Big|_{z=\frac{1}{\gamma}}$$

(10)

where $L^{-1}\{.\}$ denotes the inverse Laplace transform. Using [14 eq. (07.27.26.0004.01)], followed by [10, eq. (9.31.2)] and some algebraic manipulations, eq. (10) can also be expressed as

$$=1-L^{-1}\left\{\frac{\beta\sqrt{\pi}}{2^{M-2}\Gamma(M)}G_{4,2}^{0,4}\left(\frac{1}{4\beta s}\left|\begin{matrix}1,2,1-M,2-M\\\frac{M}{2},\frac{3-M}{2}\end{matrix}\right.\right)\right\}_{z=\frac{1}{\gamma}} \quad (11)$$

where $G[.]$ is the Meijer-G function [10]. Using the formula for the inverse Laplace transform of the Meijer-G function [11 eq. (3.38.1)], we obtain

$$F_{\gamma_{eq}}(\gamma)=1-\frac{\beta\sqrt{\pi}}{2^{M-2}\Gamma(M)}z^{-1}G_{3,2}^{0,3}\left(\frac{z}{4\beta}\left|\begin{matrix}2,1-M,2-M\\\frac{M}{2},\frac{3-M}{2}\end{matrix}\right.\right)\Bigg|_{z=\frac{1}{\gamma}} \quad (12)$$

Substituting $z=\frac{1}{\gamma}$ and using the property of Meijer-G function [10, eq. (9.31.2)], we finally get

$$F_{\gamma_{eq}}(\gamma)=1-\frac{\sqrt{\pi}}{2^{M}\Gamma(M)}G_{2,3}^{3,0}\left(4\beta\gamma\left|\begin{matrix}1+\frac{M}{2},\frac{1+M}{2}\\0,M,M+1\end{matrix}\right.\right) \quad (13)$$

To obtain pdf of γ_{eq}, we need to find derivative of eq. (13) with respect to γ. Using the properties of Meijer-G function [10 eq. (9.31.3)] and [10 eq. (9.31.2)] the pdf of γ_{eq}, $p_{\gamma_{eq}}$, is obtained as

$$p_{\gamma_{eq}}(\gamma)=\frac{\sqrt{\pi}}{2^{M}\Gamma(M)\gamma}G_{2,3}^{0,3}\left(4\beta\gamma\left|\begin{matrix}1+\frac{M}{2},\frac{1+M}{2}\\1,M,M+1\end{matrix}\right.\right) \quad (14)$$

C. Derivation of the mgf of received SNR at destination

The mgf of γ_{eq}, $M_{\gamma_{eq}}(s)=E_{\gamma_{eq}}(\exp(-s\gamma))$, with the help of [10 eq. 7.813.1] and little algebra, is given by,

$$M_{\gamma_{eq}}(s)=\frac{\sqrt{\pi}}{2^{M}\Gamma(M)\gamma}G_{3,3}^{1,3}\left(\frac{s}{4\beta}\left|\begin{matrix}1-M,0,-M\\0,-\frac{M}{2},\frac{1-M}{2}\end{matrix}\right.\right) \quad (15)$$

Using [14 eq. (07.27.26.0004.01), the above mgf can be rewritten as

$$M_{\gamma_{eq}}(s)= {}_3F_2(1,M,1+M;\frac{M+1}{2},\frac{M+2}{2};-\frac{s}{4\beta}) \quad (16)$$

It may be noted that for $M=1$, (16) simplifies to eq. (22) of [1] with $\overline{\gamma}=\frac{1}{\beta}$.

III. PERFORMANCE ANALYSIS

Using the above results for pdf, cdf, and the mgf of destination SNR one can derive the various performance measures to study the impact of the multiple-antenna relay system on overall communication link, such as the outage probability(OP), the average bit error rate(ABER) or average symbol error rate(ASER), the destination SNR moments.

The outage probability of a diversity combiner is defined as the probability that the instantaneous SNR, γ, fallsbellowsome prescribed threshold(γ_{th}).Mathematically, it is given by

$$P_{op}=\Pr\left[0\le\gamma\le\gamma_{th}\right]=F_{\Gamma}(\gamma_{th}) \quad (17)$$

Using (13) and (17), the outage probability can be shown to be given by

$$P_{op}=1-\frac{\sqrt{\pi}}{2^{M}\Gamma(M)}G_{2,3}^{3,0}\left(4\beta\gamma_{th}\left|\begin{matrix}1+\frac{M}{2},\frac{1+M}{2}\\0,M,M+1\end{matrix}\right.\right) \quad (18)$$

It can be shown that (18) can easily be reduced to[13, eq. (17)] for $M=1$. It may be noted that the lower bound of the outage probability for the fixed relay multi-antenna can be achieved by letting $M\rightarrow\infty$ [13]. Following the same reasoning as in [13], if M goes to infinity, the SNR of the S $\rightarrow$ R link can be considered very much large compare to that of the R D link and hence the overall received SNR is dictated only by the R D. The outage probability for this case can be written as,

$$P_{op}=1-e^{-\beta\gamma_{th}}$$

B. Average Bit Error Rate (ABER)

Average BER of several digital modulation schemes may be evaluated by using the MGF-based approach outlined in [12] once the MFG is known .For example, the ABER of binary differential phase-shift keying (DPSK) and noncoherent orthogonal frequency-shift keying (NCFSK) can be written in the following form

$$P_b(e)=aM_{\gamma_{eq}}(b) \quad (19)$$

where $a=0.5$ and $b=(1,0.5)$ for BDPSK and BNCFSK respectively.

C. Received SNR moment

The received SNR moment at the destination (i.e., moment of γ_{eq}), can be obtained with the aid of eq. (14) and[10 eq. 7.811.4].It is given by

$$E\left(\gamma_{eq}^{n}\right)=\frac{(M)_{n}(M+1)_{n}}{(\frac{M+2}{2})_{n}(\frac{M+1}{2})_{n}}\times\frac{\Gamma(1+n)}{(4\beta)^{n}} \quad (20)$$

where $(a)_{n}=a(a+1)\cdots(a+n-1),(a)_{0}=1,a\ne0$,is the Pochhammer symbol. For example, with n=1

$$E\left(\gamma_{eq}\right)=\frac{M}{\beta(M+2)}=\frac{M}{(M+2)}\overline{\gamma}$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

In Fig.2, eq. (18) is used to plot the outage probability curves for *amplify-and-forward (AF)* multi-antenna fixed relay (infrastructure-based relay) system with $\overline{\gamma}=\frac{1}{\beta}$. The horizontal axis represents the instantaneous SNR in dB and the vertical axis represents the outage probabilities. From

Fig. 2, it is clear that the improvement is significantly high, for M =2 (roughly 3 dB) relative to M=1 in the region where the SNR is greater than 5 dB . However, it is observed that for M>2 relative to M=2, the additional improvement achieved is little for low SNR regimes and insignificant in the high SNR regimes. For M>2 the plots quickly align with the curves for M=2 as the SNR increases. This behavior was also reported in[13] while studying the outage performance with multiple-source antennas.

Fig.3. compares the variation of the ABER for binary DPSK with respect to the average SNR per branch for infrastructure-based fixed relays over flat Rayleigh fading channel with varying number of antennas M. Similar performance is noted (as in the case of outage probability). From the figure it is clear that employing two antennas at the input improve the overall ABER both at the low and high SNR regimes. However, employing more number of antennas (M>2) at the relay input has little influence to improve the overall link performance compare to two antennas(M=2). This is due to the fact that in case of infrastructure-based fixed relays the S $\rightarrow$ R link is improved whereas overall performance is determined by both S R and R D links.

In view of the above it can be inferred that for all practical purposes employing fixed relays the best performance can be achieved when the relay has two antennas at the relay input.
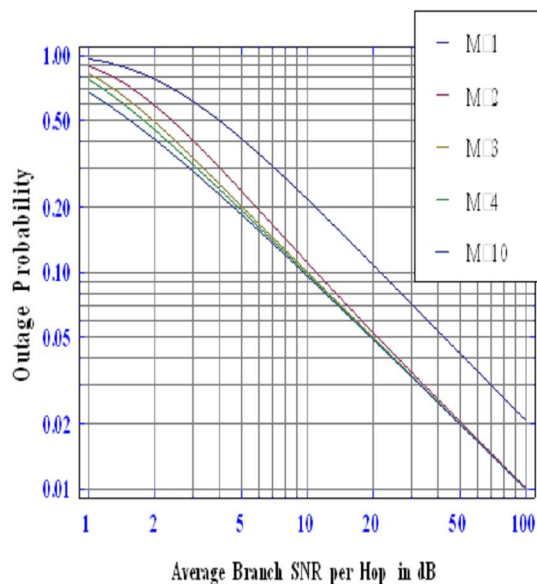

Figure 2. Comparison of Outage Probability of *amplify-and –forward* relaying with diversity antennas (M) at the relay.
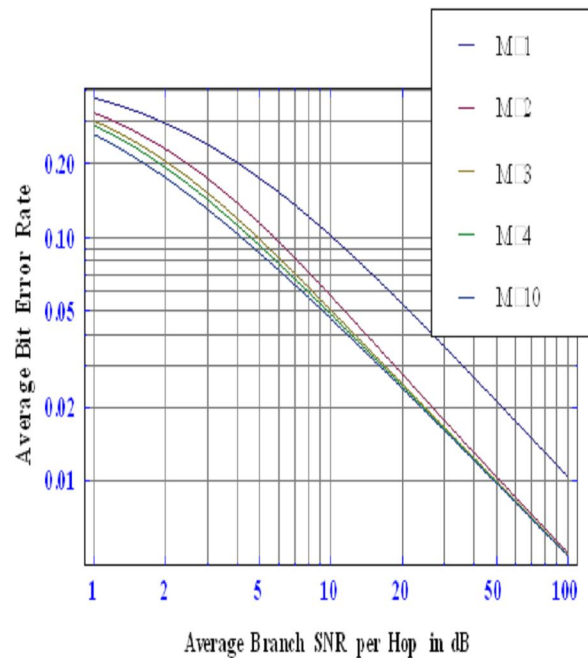

Figure 3. ABER of BDPSK for *amplify-and–forward* relaying with diversity antennas (M) at the relay over Rayleigh fading channel.

V. CONCLUSION

In this work we have presented new closed form expressions for the statistics of infrastructure-based fixed multi-antenna relays. Based on the derived expressions, the error performance of such link structure is studied analytically in terms of the ABERs and outage probabilities for *amplify-and-forward* relaying systems. It is also shown that the best performance may be achieved when the number of antennas is two at the relay input. The results can be extended for studying the performance of multi-relay cooperative diversity systems and evaluate performance of infrastructure-based multi-antenna relay in various digital receivers.

REFERENCES

[1]   M.O. Hasna and M.-S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh-fading channels," *IEEE Trans .Wireless Commun..*, vol. 2, no. 6, pp. 1126-1131, Nov. 2003.

[2]   A.Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity,part I and II," *IEEE Trans. Commun.*, vol. 51, no. 11, Nov. 2003.

[3]   J. N. Laneman, D. Tse, and G. W. Wornell, "Cooperative diversityin wireless networks: efficient protocols and outage behaviour," *IEEETrans. Inform. Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.

[4]   R. U. Nabar, H. Bolcskei, and F. W. Kneubuler, "Fading relay channels:performance limits and space-time signal design," *IEEE J. Select. AreasCommun.*, Aug. 2004.

[5]   B. Wang, J. Zhang, and A. Host-Madsen, "On the capacity of MIMO relay channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 1, pp. 29–43, Jan.2005.

[6]   A. Adinoyi and H. Yanikomeroglu," Cooperative Relaying in Multi-Antenna Fixed Relay Networks,"*IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp.533-544, Feb. 2007.

[7]   A. Dey,P. K. Ghosh and K. Gupta," Performance of Multiple Selection Diversity Antennas at the Relay Input for Relaying System," in *Proceedings of International Conference on Recent Trends in Information, Telecommunication and Computing (ITC),* 2010.

[8]   Simon Haykin and Michael Moher, *Modern Wireless Communications* , Pearson Education, 2005.

[9]   A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Process*, 4th ed. New York: McGraw-Hill,2002.

[10]   I. S. Gradshteyn and I. M. Rzyhik, *Table of Integrals, Series and Products*, 5th ed. Academic Press, 1994.

[11]   A. P. Prudnikov, Y. A. Brychkov, and O. I. Marichev, *Integral and Series:Inverse Laplace Transforms*, 5th ed. London, U.K.: Gordon and Breach,1992, vol. 5.

[12]   M. K. Simon and M.-S. Alouini, Digital Communication over Fading Channels: A Unified Approach to Performance Analysis. New York: John Wiley & Sons, 2000.

[13]   Hyunkee Min, Sungeun Lee, Kyungchul Kwak, and Daesik Hong, "Effect of Multiple Antennas at the Source on Outage Probability for Amplify-and-Forward Relaying Systems ," *IEEE Trans .Wireless Commun..*, vol. 8, no. 2, pp. 633-637, Feb. 2009.

[14]   'The Wolfram function site',[online]. Available: http:// Functions.wolfram.com.

ACEEE